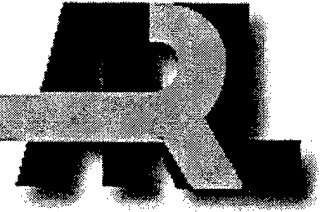


ARMY RESEARCH LABORATORY



# Frequency Estimation From Accelerometer Measurements

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## Abstract

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Measurements from angular rate sensors and accelerometers allow inertial navigation. While this report focuses on accelerometers (since they are conceptually simpler), all the discussion is directly applicable to angular rate sensors. In fact, estimation techniques were developed to analyze angular rate sensors attached to a body subjected to sinusoidal motion. System developers need to be able to predict performance of a system, based on the component specifications supplied by the manufacturers and signal processing. Given knowledge of the noise level, the time correlation of the noise, the number of wavelengths processed, and the sampling rate, a model is proposed to predict the accuracy of the estimated parameters of a sinusoidal input. It is hoped that this model will allow the designer to quickly determine the required specifications for a particular application.

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# FREQUENCY ESTIMATION FROM ACCELEROMETER MEASUREMENTS

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## 1. Introduction

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Measurements from angular rate sensors and accelerometers allow inertial navigation. While this report focuses on accelerometers (since they are conceptually simpler), all the discussion is directly applicable to angular rate sensors. In fact, estimation techniques were developed to analyze angular rate sensors attached to a body subjected to sinusoidal motion. System developers need to be able to predict the performance of a system, based on the component specifications supplied by the manufacturers and signal processing. Given knowledge of the noise level, the time correlation of the noise, the number of wavelengths processed, and the sampling rate, a model is proposed to predict the accuracy of the estimated parameters of a sinusoidal input. It is hoped that this model will allow the designer to quickly determine the required specifications for a particular application.

A simulation was developed to create data for the desired situations. Two estimates are pursued. First, an estimation technique that identifies the dominant frequency<sup>1</sup> in the signal is discussed. The second frequency estimation technique introduces new estimation criteria based on an inner product. A model of estimation error is presented, and some statistical concerns are addressed.

The frequency of the sinusoid, the power/amplitude, and the phase are the parameters of interest. Notice that when the signal of interest is known to be sinusoidal, the bias or offset from zero is of no concern. Once the frequency is found, the bias is the average of the signal over a whole number of wavelengths. It does not affect the estimates of the sinusoidal parameters. Any time an accelerometer has a sinusoidal input, the bias can be estimated. It may be possible to estimate bias for accelerometers installed in artillery rounds under the assumption that the coning motion is sinusoidal. First, an accelerometer must be aligned so that it is in the coning plane, and the bias must be calculated based on the assumption that the signal is sinusoidal. Then, in-flight calibration can be achieved via a mechanism to rotate the accelerometer into the axis of motion.

The frequency is the parameter of greatest interest. If the estimate of frequency is wrong, our predictions will deviate at the rate of the frequency estimation error. By having a "bound" on the error in frequency estimation, it is possible to determine the duration of useful prediction for an estimate. Errors in phase will cause a shift in the sinusoid and do not change with time. Similarly, estimates of

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<sup>1</sup>The dominant frequency is the frequency that yields the largest amplitude or power when the signal is projected onto it.



the power or magnitude of the sine wave will be similar to the actual wave and will not change over time.

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## 2. Simulation

---

The signal simulation is in SIMULINK®, an icon-based software product that allows system analysis. An accelerometer model was developed which consisted of the input, a bias added to the input, and a time-correlated noise process added to the input. A SIMULINK® block for a time-correlated noise process was developed for the accelerometer model. This simple model is justified if the accelerometer is assumed to have been calibrated. Scale factor corrections and predetermined biases are assumed to be properly accounted. The model represents all the un-modeled biases as a single bias and adds noise to the signal. A correlated noise process is used. Noise is correlated if the current noise value depends on the value in the recent past. The time correlation indicates the duration of the dependence. A first order differential equation can be used to model correlated noise.

The input to the accelerometer was chosen to be a sinusoidal function. By setting the parameters of the simulation noise level, time correlation, and step size, one can generate the desired data sets. Step size will determine the number of points generated per unit time. This simulation did not represent the internal workings of the accelerometer. The model is shown in Figure 1, and Figure 2 shows the accelerometer block. Output from the model includes the acceleration, velocity, and distance. The model blocks labeled  $1/s$  are integrators.

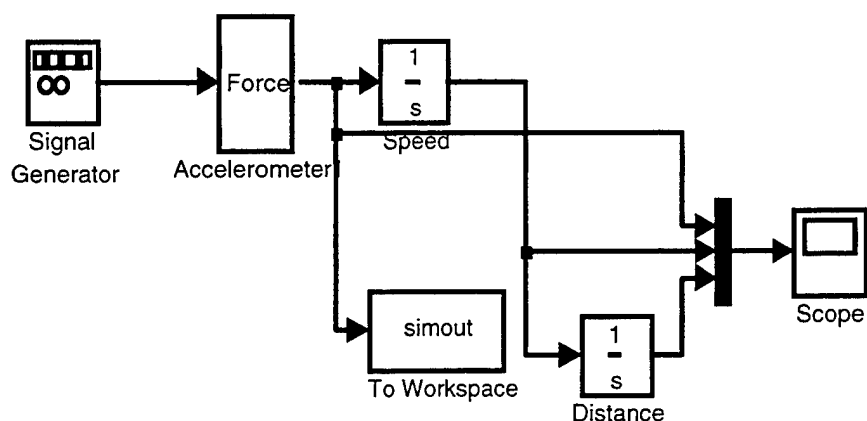


Figure 1. Signal Generation Model.

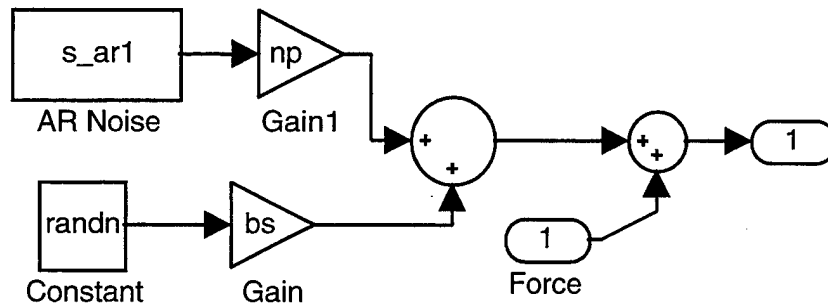


Figure 2. Accelerometer Model.

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### 3. Signal Processing

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In this case, both the power spectrum and a Fourier analysis are not that helpful since the quantities of interest are the parameters of the sinusoid with the greatest power. In Fourier analysis, an orthogonal basis of sinusoids is used to represent the signal. This set will capture all the energy of the signal; however, it will usually miss the actual frequency of the signal. The set of orthogonal sinusoids is discrete and if the frequency of interest falls in between the Fourier frequencies, it is difficult to estimate precisely.

A set of routines was developed to find the frequency of maximum power. The method used is similar to either an ideal receiver or the method used for wavelets. Wavelets use an orthogonal set of functions to represent a signal. They are useful when a few of the wavelets can capture most of the information in the signal. Generating a good set of orthogonal functions for a particular problem can be a difficult task. In this case, the only information of interest is that associated with the sinusoid of the greatest power; thus, all other information in the signal can be ignored. Since the signal is known to be sinusoidal, the time delay and amplitude can be ascertained by projections of the signal onto sine and cosine waves. Note that different frequencies require different time intervals in order to maintain the orthogonality of the sine and cosine functions. First, a routine to find the power and phase of a specified frequency was developed. This routine was then used to map the power surface within a region of the frequency space. It was observed that power, as a function of frequency, was a smooth function in the region of maximum power. An iterative search routine using a quadratic estimator was designed to find the frequency of maximum power.

A second estimation method for frequency was also devised. This method is based on the observation that sine and cosine functions of the desired frequency are orthogonal. The inner product of the signal projected onto the sine and cosine will be zero if the proper frequency is selected. By calculating these quantities for different frequencies, one can ascertain the desired frequency. The estimation process then becomes a zero crossing problem. This estimation criterion will be referred to as the "inner product criterion." As in the previous case, only frequencies completing a whole number of cycles over a selected data set could be directly evaluated.

Methods to select the parameter values can be based on other criteria. Usually, a criterion that is a function of the residuals is selected. The parameters can be chosen so that the sum of the residuals is zero. Minimization of the H1, H2, or H-infinity norms of the residuals can be achieved. These norms correspond to the sum of the absolute value of the residuals, the sum of the squares of the residuals, and the maximum residual.

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#### 4. Performance Model

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In the model, estimation error is assumed to be a function of noise level, time correlation, number of wavelengths processed, and the number of data points per wavelength. The error is expected to be infinite if no datum is processed; thus, at levels of zero for both wavelengths processed and number of data points, the predicted error should be infinite. Error is expected to be zero if there is no noise in the accelerometer; thus at zero noise level, the estimation error should be zero. A multiplicative model with unknown exponents was considered to be of the correct form. Negative exponents will indicate that the error becomes smaller when the value of an independent variable increases, while positive exponents will cause the error to increase as the magnitude of an independent variable increases. The number of wavelengths and samples per wavelength should have negative exponents, while the noise level should have a positive exponent. These assumptions can be used as a logical verification of the model after the estimation process. For this case, least squares can be used to estimate the exponents of the independent variables via the log transformation of the estimation error (y) as the dependent variable. Using Xs to represent the independent variables, the model

$$y = c x_1^\alpha x_2^\beta x_3^\delta x_4^\phi \varepsilon$$

transforms to the model

$$\ln(y) = \ln(c) + \alpha \ln(x_1) + \beta \ln(x_2) + \delta \ln(x_3) + \phi \ln(x_4) + \ln(\varepsilon) .$$

Note for this model that if the dependent variable were raised to a power, this would result in a multiplication of each of the estimated parameters. Typically, for a statistical interpretation to be appropriate, the residual error term should

have a Gaussian distribution. Statistically, the goal is to estimate the average error and be able to make a statement of the form "95% of the errors are less than X for these values of the independent variables."

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## **5. Statistical Issues**

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The model will be used in the original domain. In the log domain, small errors have large (negative) magnitudes, while large errors have smaller magnitudes. A least squares estimation will tend to give more influence to numbers of greater magnitude; however, it is desirable for the model to more accurately predict large errors. When the log model is transformed into the original domain, the sum of the residuals is no longer zero. This can be corrected by adjusting the parameters using a Newton-Raphson method to find the proper value of the parameters. Another approach is to minimize a function of the residuals. Typically, the sum of the absolute values, the sum of the squares, or the greatest deviation is considered as a possible metric to be minimized. A collection of routines was developed to minimize the selected metric.

After the parameters are adjusted to minimize a criterion, a chi-square test can be used to test the fit of the model. Through the formation of the ratio of the squared estimation error over the model-predicted variance, a chi-square random variable is formed. By summing these and comparing the sum to the proper critical values, one can test the null hypothesis (i.e., that the proposed model is adequate for the observed data). Note that the model derivation is independent of the chi-square test. Another possibility would be to search for the parameters that give the best fit to a chi-square distribution (50% level). This latter method would be a form of maximum likelihood estimation.

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## **6. Discussion of Estimation Criteria**

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The inner product criterion and the maximum power criterion for estimating the frequency were compared. Over the region observed, the power curve had a quadratic component (see Figure 3); however, increasing the amount of data or number of wavelengths processed also increased the curvature of the power curve. This increased curvature is not modeled well by quadratic functions; higher order estimators become necessary. As shown by Figure 4, the inner product plot appears to be sinusoidal and has a steep slope when it crosses zero. The linear model for the inner product criterion remains valid as the number of wavelengths processed increases. For the inner product criterion, an increase in the number of wavelengths processed increases the slope as it crosses zero. Noise tends to flatten the curves for both criteria; however, the steep slope of the inner product

criterion is more robust. Perturbations because of noise should result in a lower estimation error if the inner product criterion is used.

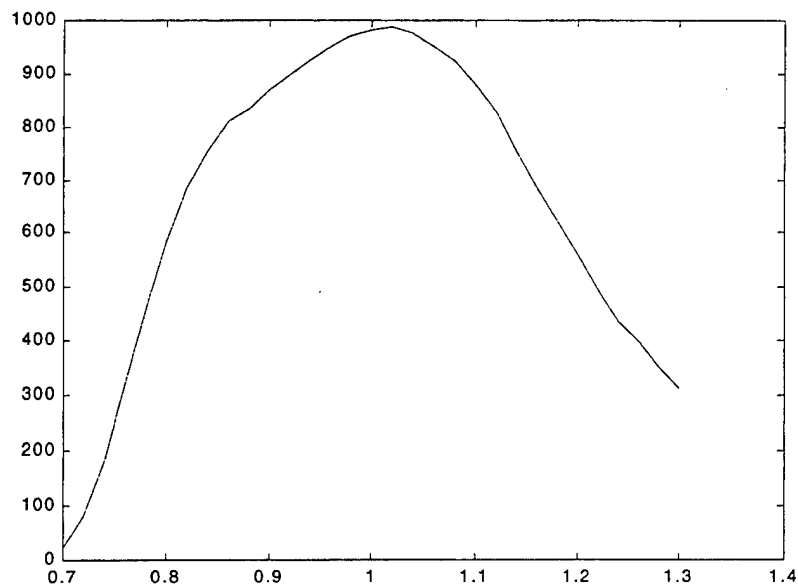


Figure 3. Power Curve.

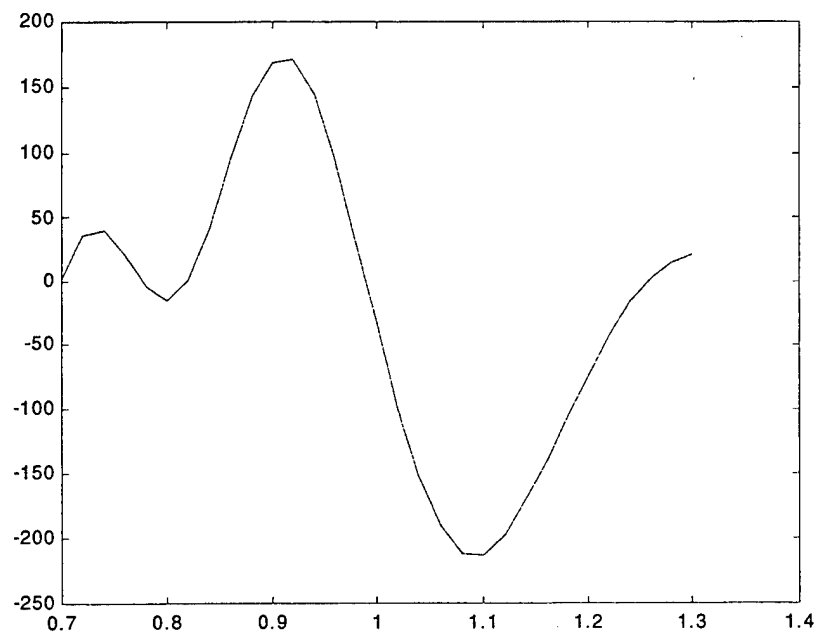


Figure 4. Inner Product Curve.

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## 7. Bias

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Even without the addition of a deterministic bias, a bias is present in the data. There are two elements that cause this undefined bias. First, the phase of the signal at the beginning of processing will cause a small but persistent offset. Suppose the accelerometer signal is a sine wave. The integration of the signal over the first 180 degrees yields the maximum positive value, and integration over the next 180 degrees returns the result to zero; thus, the velocity term is always positive. See Figure 5 for assistance in visualizing the deterministic bias. The integral of the velocity will then digress from the true value. Through this argument, it is evident that the phase of a sinusoidal signal will cause a bias in its integral. Next, recall that the integration operator is known to be linear. The integral of a signal plus noise can be evaluated from the sum of the integral of the signal (as previously discussed) and the integral of the noise. The integral of noise is a random walk. A random walk has an equal chance of increasing or decreasing at any point in time; the best estimate of its future value is its current value. Any offsets attributable to the random walk cause a bias in the velocity term. This component of the bias cannot be predicted *a priori* and will change as a function of time. System performance will be determined by how well the effects of this random walk can be minimized. High levels of noise and long time correlation increase the magnitude of the bias caused by a random walk. Figures 6 and 7 provide examples.

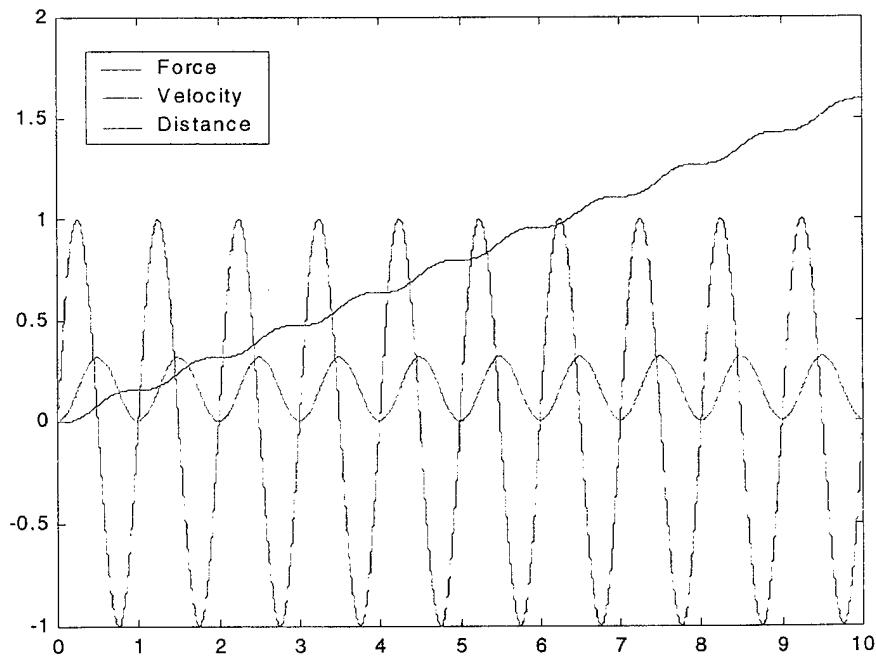


Figure 5. No Noise Response.

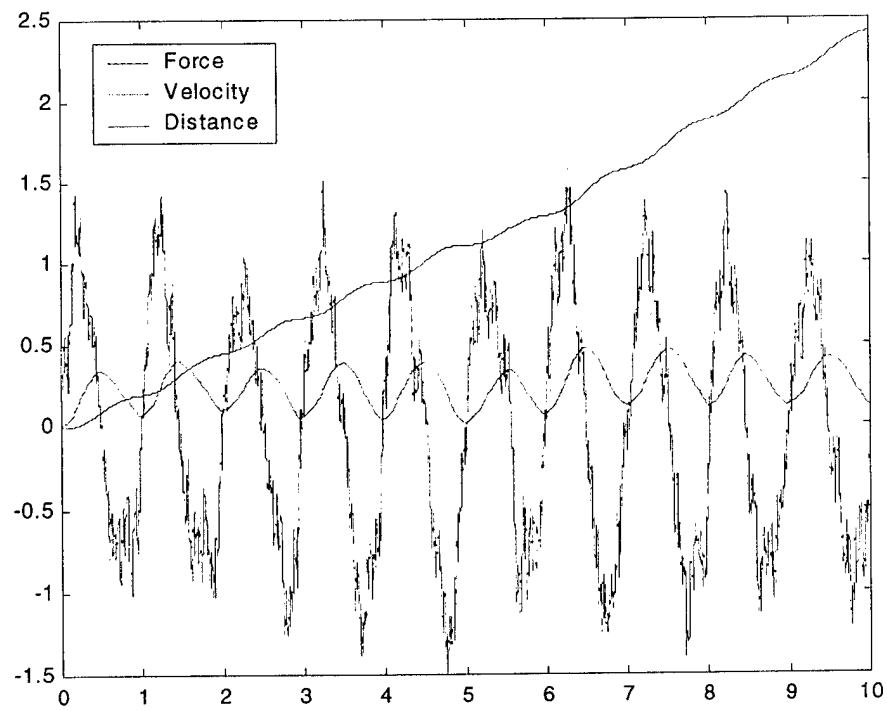


Figure 6. Low Noise Condition.

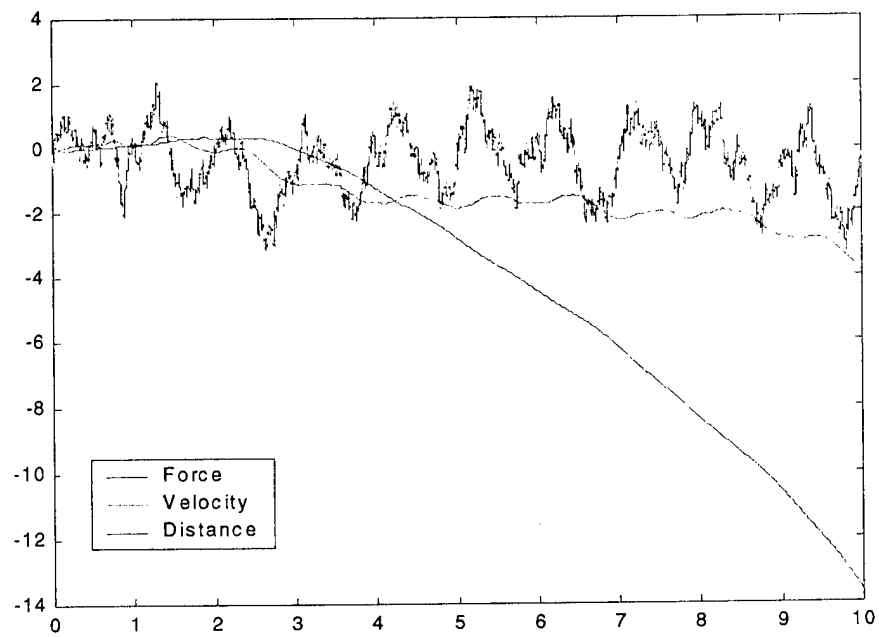


Figure 7. High Noise Condition.

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## 8. Discussion of Variables

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The four variables considered (number of points per wavelength, number of wavelengths, noise strength, and time constant of the noise) were investigated in ranges considered to be useful for inertial navigation.

Although it was possible to estimate frequency with ten observations per cycle, the estimation methods were not always stable in this region. At 20 observations per wavelength, the estimation methods were stable, and it was apparent that an increase in the number of observations beyond 20 per wavelength did not improve the estimation fidelity. This led to the conclusion that the variable (observations per wavelength) was asymptotic and had reached a near maximum value around 20 observations per wavelength. In the future, the estimation routines could be enhanced for better performance in regions of few observations per wavelength.

The time constant of the noise process indicates the auto-correlation time of the noise. At high noise levels, the effect of a long time constant is to decompose the sinusoid into segments and increase the bias in the random walk, making it difficult to estimate any of the sinusoidal parameters precisely. Based on observations of accelerometer and angular rate sensors, the time correlation is small. Using a time correlation of 0.01 or larger had no discernible effects on parameter estimation. The model for the time constant is a first order differential equation. In some cases, the time constant can be thought of as a means of removing the effects of un-modeled differential equations.

The variables accounting for the noise level and the number of wavelengths processed were retained in the model. The final model containing these three parameters is

$$y = c X_1^\alpha X_2^\beta \varepsilon$$

Notice that the noise ( $X_1$ ) should have a positive exponent, and as the noise value goes to zero, the error should also approach zero. The number of wavelengths ( $X_2$ ) should have a negative exponent. The error should approach infinity as the number of wavelengths goes to zero. This provides for interesting behavior around the point (0,0). Fortunately, the region of interest does not contain this point. Obviously, the model is only considered for positive values of noise and number of wavelengths processed. Figure 8 is included to help the reader visualize the model.



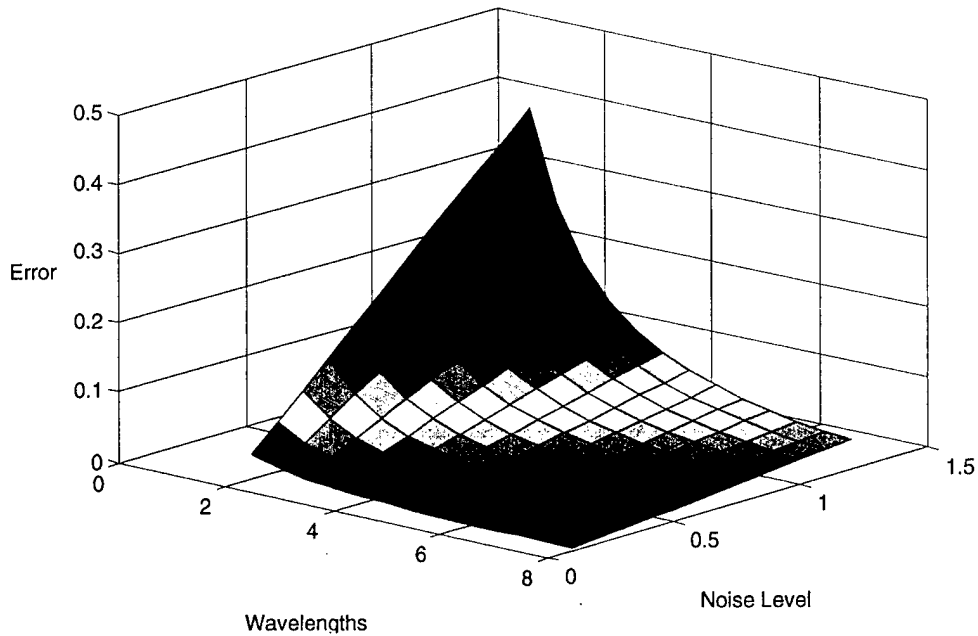


Figure 8. Error Function.

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## 9. Fitting Model Parameters

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Using the accelerometer model, we collected 60 observations. Noise levels were set in relation to the sinusoid amplitude. At noise levels of 0.2, 0.4, 0.8, and 1.2, the estimation routine processed two and four wavelengths of data. For the 0.8 and 1.2 noise levels, six wavelengths of data were also processed. Six observations were collected for each pair of conditions.

In the first part of the fitting process, the model was transformed to the log domain, and regression provided the initial parameter estimates. Two routines were devised to search the parameter space. The first routine calculates the H1, H2, H-infinity norms, and the sum of the residuals. The second routine calculates the p-value for a chi-square test, based on the assumption that the model gives the standard deviation of the estimation process. With this software, a heuristic search method was used to find a set of parameters that were acceptable for both the norm and chi-square criteria.

The exponent of the noise term was determined to be close to 1 and was set to 1. The model then is linear along the noise dimension. The exponent of the variable for wavelength was  $-5/3$ . The decrease in error along the wavelength dimension is dramatic. The third parameter is a factor that adjusts the overall height of the surface; its value is 0.1083.

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## 10. Model Testing

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Two assumptions were used to test the model. First, estimation errors were assumed to have a normal distribution. Next, the model was assumed to be true. The value given by the model was the standard deviation of the error for that point. Under these assumptions, the observed error over the model standard deviation forms a chi-square variable with one degree of freedom when it is squared.

The model acceptance test consisted of 20 observations. The noise values were randomly chosen from the range 0.1 to 1.2; the number of wavelengths was a randomly selected integer from the interval (2,7). The chi-square variables were formed and summed. The ensuing chi-square test with 20 degrees of freedom gave a p-value of .38 for the model. The p-value is the probability, when the assumptions are true, that if the test were repeated, the new test value would exceed the observed value. The observed value is in the region where we fail to reject the assumptions. Failure to reject the assumptions does not imply that the model is correct—only that based on the data, a rare event did not take place. For engineering purposes, the model was accepted as adequate for predicting the standard deviation of the frequency estimator.

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## 11. Concluding Remarks

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Accelerometer bias was shown to have both a deterministic and random component. This is in addition to biases caused by electronics, un-modeled temperature effects, and so forth, and indicates that despite all efforts to control or calibrate an accelerometer, there will be a random bias caused by the random walk effect of the integrated noise. This random walk effect is statistically proportional to the strength of the noise. In stressful environments with lots of vibration, it may be a major factor in accelerometer bias.

Although this investigation focused on frequency estimation, the same procedure can be used to estimate amplitude and phase errors. These models would be of the same form. Although the exact same analysis is pertinent for angular rate sensors, the accuracy of the amplitude estimate is extremely important in the interpretation of angular rate measurements.

The model validation technique is an application of statistical theory to quantify the performance of a model. The area of model evaluation is important to all areas of research. The technique developed here should be investigated in more detail. The current use, while logically correct, reverses the traditional role of the null hypothesis.

In this case, the signal was a sinusoid corrupted by noise. The approach taken was to project the signal onto sinusoids and search the parameter space to find the best set of parameters. Two methods were used to define "best"; the first was to use maximum power, and the second method used an inner product criterion. In situations when the researcher knows the functional form of the signal, the signal can be projected onto that function and the desired parameters can be defined as those that yield the most power. In this case, the researcher can pretend that he or she has a set of wavelets, but only the first one will have useful information, allowing the others to be ignored.

With flight data, all frequencies change during the flight. Frequency can be tracked via either of the estimation techniques discussed. If a model of the frequency change were available, a Kalman filter could be made to increase tracking fidelity. In many applications requiring frequency estimation, such as radar, banks of filters are used, and the filter with the greatest output is assumed to have the correct frequency. Both frequency estimation techniques mentioned here can be implemented to improve performance and reduce the number of filters required.

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